Extra Questions for Bonus
Math 342

by

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1. Prove Theorem 3.6: *For each a in a group G, the centralizer of a is a subgroup of G*, i.e $C(a) = \{g \in G | ga = ag\}$ *is a subgroup of G.*

2. In a group G every element has an inverse implies that the equations $gx = h$ and $xg = h$ are always solvable for x in G. Moreover, for $g, h \in G$ the equation $gx = h$ has a unique solution $x = g^{-1}h$ and the equation $xg = h$ has a unique solution $x = hg^{-1}$ in G. Using this fact, prove that every row and column of the cayley table of a group G contains every element of G exactly once.

3. If ϕ is an isomorphism from a group G to a group $\overline{G}$, prove that

   (a) $\phi^{-1} : \overline{G} \to G$ is an isomorphism.

   (b) If K is a subgroup of G, then $\phi K = \{\phi k | k \in K\}$ is a subgroup of $\overline{G}$.

4. Prove that $\phi_a(x) = axa^{-1}$ for all $x \in G$ is an automorphism of G, the *Inner Automorphism induced by a.*

5. prove that for a group G, $(Aut(G), \circ)$ is a group, where $Aut(G) = \{\phi | \phi : G \to G$ is an isomorphism$\}$ and $\circ$ is the functions composition.

6. prove that for a group G, $(Inn(G), \circ)$ is a group, where $Inn(G) = \{\phi_a | a \in G\}$, the set of Inner automorphisms induces by elements of G.